

Comments on “Characteristic Curves for Deep Circular Tunnels in Poroplastic Rock” by A. Bobet

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1 Introduction

Bobet (2009), hereafter referred to as “the Paper”, revisits the classic problem of the ground response curve by providing analytical solutions for the short-term and long-term responses of a porous medium with an elasto-plastic behaviour. Following the example of a number of earlier research works on poro-plastic behaviour (Lembo-Fazio and Ribacchi 1984; Giraud 1993; Wang and Dusseault 1994, 1995; Giraud et al. 2002), the Paper considers the general case of a non-unit Biot constant α . Furthermore, the assumed constitutive equations include the case of a sudden reduction in strength immediately after failure (brittle behaviour). This assumption represents a special case of the more general softening or hardening behaviour that has already been analysed by Lembo-Fazio and Ribacchi (1984), Izquierdo and Romana (1987), Wang and Dusseault (1994, 1995), Giraud et al. (2002).

Besides two major problems concerning long-term and short-term behaviour, which will be discussed in the following two sections in detail, the Paper contains some errors and inaccuracies which make it difficult to follow the mathematical derivations. For example, the initial effective stress σ_0' should appear on the right hand side of Eq. 6 and, more specifically, the term $(-1 + 2\nu) \sigma_0'$ is missing within the brackets; the Abstract characterises the material behaviour as “elastic-perfectly plastic with brittle failure” which is a contradiction in itself; the parameter ϕ introduced after Eq. 14 for characterising the associated flow rule should be probably replaced by the residual friction

angle ϕ^r ; the short-term condition is characterised, strictly speaking, by a constant fluid *mass* rather than by a constant fluid *volume*; Sect. 3 introduces one parameter and three equations (n_p , Eqs. 11, 16 and 17) which are absolutely unnecessary for the definition of the constitutive model in the present case.

2 Long-Term Solution

Section 3 of the Paper distinguishes two zones around the opening (an inner, plastic region and an outer, elastic region). The stress state at each point within the plastic region ($r < r_p$) fulfils the yield condition with residual strength values (Eq. 13a), while in the elastic region (including the elasto-plastic boundary, i.e. for $r \geq r_p$) Eq. 13b applies with the sign “ \geq ” instead of “ $=$ ”. It seems that the Paper takes for granted that no plastic strains occur in the outer zone $r \geq r_p$. This is true only in the exceptional case where a complete drainage and pore-pressure relief in the ground ahead of the face is carried out in advance of the excavation (Amberg 2009; Anagnostou 2009). For the standard case analysed in the Paper, i.e. excavation without advance drainage, the assumption made in the Paper is not correct and leads to an underestimation of the deformations (Anagnostou 2009).

The reasons for this have been widely recognized for some time (Giraud et al. 1993; Benamar 1996; Benamar and Rousset 1998; Graziani and Ribacchi 2001). The evolution of the stress field during the transient process that takes place before reaching the long-term conditions is complex: due to the pore-pressure gradients, which are temporarily very high, plastic loading and yielding takes place in a first stage, followed by elastic unloading in the final stages of the time-dependent process. The

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consequence is that three rather than two zones develop around the opening: an inner zone where the strains are partially plastic and the stresses fulfil the yield condition; a second zone, where the strains are also partially plastic but the stress states are within the elastic domain; and an outer elastic zone.

3 Short-Term Solution

The computational results presented and discussed in the Paper for the case of a non-unity Biot coefficient ($\alpha = 0.75$) are erroneous. This is demonstrated in a discussion of the plausibility and the consistency of the results published in Figs. 7, 9 and 10 of the Paper.

The starting point for the present discussion is Fig. 7 and, more specifically, the discontinuity in the effective stresses at the elasto-plastic interface. The Paper notices this jump (*it is interesting to note that for the short-term case there is a jump in effective stresses at the plastic–elastic boundary*) and attempts to explain it by making reference to the equilibrium condition and to the discontinuous pore-pressure distribution of Fig. 9 (*... since equilibrium requires that the radial stresses are continuous across the boundary, a jump in the pore pressures carries a jump in the effective stresses*). This statement is in itself correct, but nevertheless does not explain why the pore pressure Δu should experience such a jump.

The short-term pore-pressure drop within the plastic zone is a well-known effect (cf., e.g. Mair and Taylor 1993; Giraud et al. 1993). What is unexpected in the present case is the pore-pressure discontinuity. As can be seen from Eq. 5, under the undrained conditions governing the short-term response (i.e. for $\zeta = 0$), a pore-pressure jump Δu is associated with a jump in the volumetric strain $\Delta \varepsilon_{\text{vol}}$. Since the volumetric strain is equal to the sum of radial and tangential strain and the latter cannot experience a jump (because it is equal to U_r/r and the radial displacement U_r must be continuous), a pore-pressure jump is associated with a jump in the radial strain dU_r/dr , i.e. the $U_r(r)$ curve should have a kink at $r = r_p$ and become abruptly steeper in the plastic region. However, Fig. 10 shows exactly the contrary. At $r/r_0 = 2$ (i.e. at the location of the elasto-plastic interface according to Fig. 9), the slope in the plastic region is surely not higher than in the elastic region. The computed pore-pressure jump is therefore not consistent with the computed deformations.

The second inconsistency concerns the calculated effective stresses in the plastic region (Fig. 7). In the computational example of Fig. 7, the material is perfectly plastic ($\phi^r = \phi^p = 15^\circ$). Consequently, the jump in the tangential effective stress observed in Fig. 7 at the elasto-plastic interface (at about $r/r_0 = 2$) is associated with the

jump in the radial effective stress as the yield condition (Eq. 13) requires that $\Delta \sigma_\theta' = N_\phi \Delta \sigma_r' = 1.7 \Delta \sigma_r'$. Figure 7 shows, however, that the jump in the effective tangential stress $\Delta \sigma_\theta'$ amounts to about $2.2 \Delta \sigma_r'$, which is clearly more than the yield condition would allow.

Finally, the presented results violate the equilibrium condition. More specifically, the total radial stress does not satisfy the boundary condition at the tunnel wall and at the elasto-plastic interface (i.e. the first and the fourth conditions given by Eq. 18). The solid curve in Fig. 1a of the present discussion shows the distribution of the total radial stress. It has been calculated on the basis of Eq. 2 (with $\alpha = 0.75$) and the results (pore pressure and effective radial stress) of Figs. 7 and 9. The total radial stress exhibits a discontinuity at about $2r_0$. Furthermore, the support pressure amounts to about 0.8 MPa, which is considerably higher than the prescribed boundary pressure ($\sigma_i = 0.5$ MPa).

We checked whether it is possible to reproduce the computational results of the Paper by applying the equations presented and the iterative procedure proposed for calculating the plastic radius on the basis of Eq. 24. We obtained similar results as in Figs. 7, 9 and 10, but the computed plastic radius amounts to $0.23r_0$, i.e. it is smaller than the tunnel radius which cannot be true. The plastic radius indicated by Figs. 7 and 9 (i.e. $2r_0$) does not satisfy Eq. 24.

The reason for these inconsistencies seems to be an error in the last term of the equation for the constant C_1 (Eq. 23). The last operation must be a multiplication rather than an addition (else the dimensions would be false). The corrected last term of this equation reads as follows:

$$C_1 = \dots \left[\frac{E}{(1+\nu)A} (s_2 + N_\psi) + (s_2 + 1)(\alpha^r)^2 M \right] \Bigg\}.$$

The dashed curves in the Fig. 1 of the present discussion show the distribution of the total radial stress, of the effective radial stress and of the pore pressure, respectively, computed by applying the corrected equation. The solid curves have been taken from the Paper and are given for the purpose of comparison. The results obtained from the corrected equation do not exhibit the inconsistencies of the published data. The boundary conditions are satisfied and there is no jump.

4 Conclusions

The long-term solutions that are presented in the Paper apply only to the exceptional case where advance drainage is carried out and a complete pore-pressure relief takes place in the ground ahead of the face. The short-term computational results for $\alpha = 0.75$ are based upon an

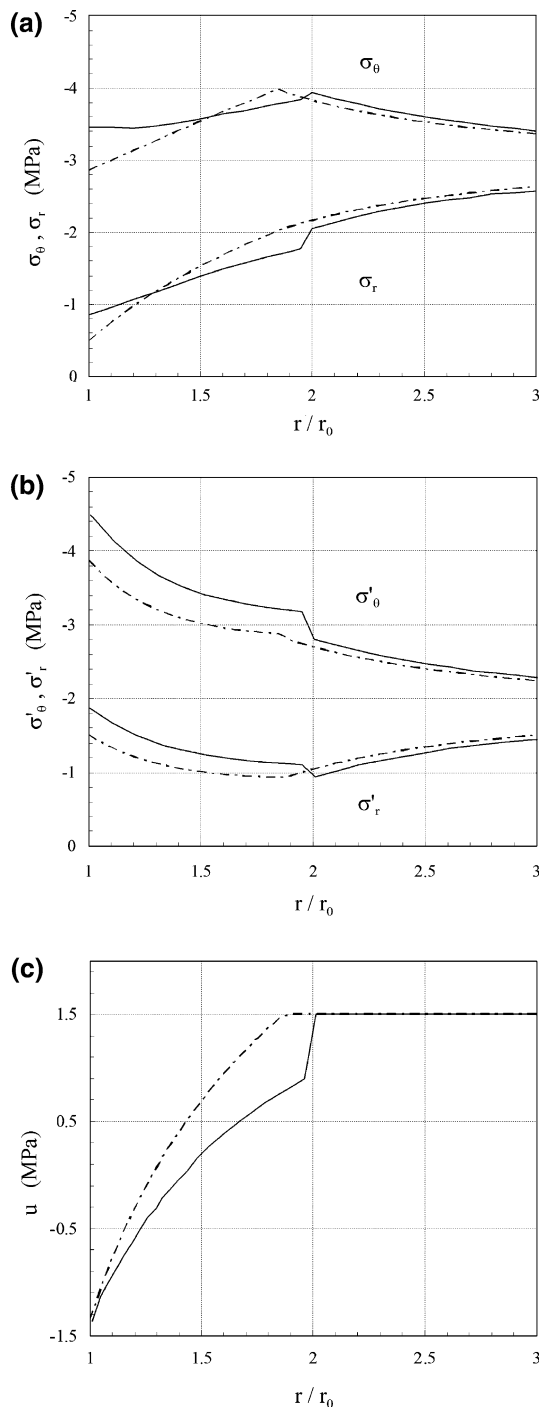


Fig. 1 Short-term distribution of **a** the total stresses, **b** of the effective stresses and, **c** of the pore pressure (solid lines after Bobet 2009, dashed lines computed based upon corrected equation)

erroneous equation. It is unfortunate that the author did not notice the obvious inconsistencies in the computational results (such as the violation of the boundary conditions) or attempted to provide ambitious explanations for implausible results (such as the pore-pressure discontinuity).

We would like to conclude this discussion with a mention of some of the original research results that have been obtained in this field in the past. For example, the deformation-dependency of the permeability coefficient, which is briefly mentioned in Sect. 3 of the Paper, was taken into account long ago in closed-form poroplasticity solutions for brittle rock (cf., e.g. Lembo-Fazio and Ribacchi 1984; Graziani and Ribacchi 2001). The same can be said about the effects of the stress path dependency of the material behaviour (mentioned in the Paper Sect. 5). Giraud et al. (1993), Benamar (1996) and Graziani and Ribacchi (2001) deal precisely with this question. The work of Benamar (1996) is particularly interesting in the context of the Paper, as he considered not only the short-term and the long-term conditions but also presented analytical solutions for the complete transient, poro-plastic problem.

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